

الباب الثاني عشر

امثلة متنوعة

Various Problems

$$\lim_{x \rightarrow 0} \frac{1 - \sin x}{\cos x}$$

مثال: أحسب النهاية

الحل: بوضع

$$L = \lim_{x \rightarrow 0} \frac{1 - \sin x}{\cos x} = \frac{1 - 0}{1} = 1 \begin{cases} \because x \rightarrow 0 \\ \therefore x \neq 2 \end{cases}$$

$$\lim_{x \rightarrow \pi/4} (1 - \cot x)$$

مثال: أحسب النهاية

الحل: بوضع

$$L = \lim_{x \rightarrow \pi/4} (1 - \cot x)$$

$$= 1 + \cot(\pi/4) = 1 + 1 = 2 \begin{cases} x \rightarrow \pi/4 \\ \therefore x \neq \pi/4 \end{cases}$$

$$\lim_{x \rightarrow 0} \frac{1 + \sin x}{1 + \cos x}$$

مثال: أحسب النهاية

الحل: بوضع

$$L = \lim_{x \rightarrow 0} \frac{1 + \sin x}{1 + \cos x} = \frac{1 + \sin 0}{1 + 1} = 1 \begin{cases} \because x \rightarrow 0 \\ \therefore x \neq 0 \end{cases}$$

$$\lim_{x \rightarrow 0^+} \cot x$$

مثال: أحسب النهاية

الحل: بوضع

$$L = \lim_{x \rightarrow 0^+} \cot x = \lim_{x \rightarrow 0^+} \frac{\cos x}{\sin x} = 1$$

$$\sin x \rightarrow 1 \text{ and } \cos x \rightarrow 0 \text{ as } x \rightarrow 0 \text{ and } x > 0$$

وبالتالى لها خط تقريبي عند $x = 0$

$$\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{\sin^2 x}$$

مثال: أحسب النهاية

الحل: بوضع

$$L = \lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{\sin^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x + \cos^2 x)}{1 - \cos^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x + \cos^2 x)}{(1 - \cos x)(1 + \cos x)} \begin{cases} \text{as } x \rightarrow 0, x \neq 0 \\ \therefore \cos x \neq 1, \cos x - 1 \neq 0 \end{cases}$$

$$= \frac{1 + \cos 0 + \cos^2 0}{1 + \cos 0} = \frac{1 + 1 + 1}{1 + 1} = \frac{3}{2}$$

$$\lim_{x \rightarrow \pi/4} \frac{\csc^2 x - 2}{\cot x - 1}$$

مثال: أحسب النهاية

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow \pi/4} \frac{\csc^2 x - 2}{\cot x - 1} \dots [\csc^2 x = 1 + \cot^2 x] \\ &= \lim_{x \rightarrow \pi/4} \frac{\csc^2 x - 1}{\cot x - 1} \\ &= \lim_{x \rightarrow \pi/4} \frac{(\cot x + 1)(\cot x - 1)}{\cot x - 1} \end{aligned}$$

$$\begin{aligned} L &= \lim_{x \rightarrow \pi/4} (\cot x + 1) \dots [\text{as } x \rightarrow \pi/2; x \neq \pi/4 \rightarrow \cot x \neq 1, \cot x - 1 \neq 0] \\ &= 2 \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{x}$$

مثال: أحسب النهاية

الحل: بوضع

$$L = \lim_{x \rightarrow 0} \frac{\sin 7x}{x}$$

بضرب كل من البسط والمقام في 7

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \frac{\sin 7x}{x} \\ &= \left(\lim_{x \rightarrow 0} 7 \right) \cdot \lim_{x \rightarrow 0} \frac{\sin 7x}{7x} = 7 \cdot 1 = 7 \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 8x}{\sin 5x}$$

مثال: أحسب النهاية

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \left(\frac{\sin 8x}{\sin 5x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin 8x}{\sin 5x} \cdot 8x \right) \cdot \left(\frac{5x}{\sin 5x} \cdot \frac{1}{3x} \right) = \frac{3}{2} \end{aligned}$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^3$$

مثال: أحسب النهاية

الحل: بوضع

$$L = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^3 = \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^3 = 1^3 = 1$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{\sin^2(x/2)}{4x^2}$$

الحل: بوضع

$$\begin{aligned}
 L &= \lim_{x \rightarrow 0} \frac{\sin^2(x/2)}{4x^2} \\
 &= \frac{1}{4} \lim_{x \rightarrow 0} \frac{\sin^2(x/2)}{x^2} = \frac{1}{4} \lim_{x \rightarrow 0} \left(\frac{\sin(x/2)}{x/2} \right)^2 \\
 &= \frac{1}{4} \left(\lim_{x \rightarrow 0} \frac{\sin(x/2)}{x/2} \times \frac{1}{2} \right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}
 \end{aligned}$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{\sin^2(-11x)}{\tan 9x}$$

الحل: بوضع

$$\begin{aligned}
 L &= \lim_{x \rightarrow 0} \frac{\sin^2(-11x)}{\tan 9x} \\
 &= \lim_{x \rightarrow 0} \left[\frac{\sin(-11x)}{-11x} \right] \times \left[\frac{9x}{\tan 9x} \times \frac{1}{9} \right] \\
 &= -11 \lim_{x \rightarrow 0} \left[\frac{\sin(-11x)}{-11x} \right] \times \frac{1}{9} \lim_{x \rightarrow 0} \left[\frac{9x}{\tan 9x} \right] \\
 &= -\frac{11}{9} \left[\lim_{x \rightarrow 0} \frac{\sin(-11x)}{-11x} \right] \times \frac{1}{9} \left[\lim_{x \rightarrow 0} \frac{9x}{\tan 9x} \right] \\
 &= -\frac{11}{9} [1][1] = -\frac{11}{9}
 \end{aligned}$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{\sin 3x \sin 5x}{7x^2}$$

الحل : بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \frac{\sin 3x \sin 5x}{7x^2} \\ &= \frac{1}{7} \lim_{x \rightarrow 0} \left[\frac{\sin(3x)}{x} \right] \times \left[\frac{\sin(5x)}{x} \right] \\ &= \frac{1}{7} \left[\lim_{x \rightarrow 0} \frac{\sin(3x)}{x} \right] \times \left[\lim_{x \rightarrow 0} \frac{\sin(5x)}{x} \right] \\ &= \frac{1}{7} \left[\lim_{x \rightarrow 0} \frac{\sin(3x)}{x} \right] \times \left[\lim_{x \rightarrow 0} \frac{\sin(5x)}{x} \right] \\ &= \frac{1}{7} \left[\lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \times 3 \right] \times \left[\lim_{x \rightarrow 0} \frac{\sin(5x)}{5x} \times 5 \right] \\ &= \frac{1}{7} (1 \times 3)(1 \times 5) = \frac{15}{7} \end{aligned}$$

مثال : أحسب النهاية

$$\lim_{x \rightarrow 3} \frac{\sin(4x^2 - 3x)}{x^2 - 9}$$

الحل : بوضع

$$\begin{aligned}
L &= \lim_{x \rightarrow 3} \frac{\sin(x^2 - 3x)}{x^2 - 9} \\
&= \lim_{x \rightarrow 3} \frac{\sin[x(x-3)]}{(x-3)(x+3)} \\
&= \lim_{x \rightarrow 3} \left[\frac{\sin[x(x-3)]}{[x(x-3)]} \right] \times \left[\frac{x}{x+3} \right] \\
&= \left[\lim_{x \rightarrow 3} \frac{\sin[x(x-3)]}{[x(x-3)]} \right] \times \left[\lim_{x \rightarrow 3} \frac{x}{x+3} \right] \\
&= \left[\lim_{x \rightarrow 3} \frac{\sin[x(x-3)]}{[x(x-3)]} \right] \times \left[\frac{3}{3+3} \right]
\end{aligned}$$

نفترض ان

$$x(x-3) = \theta \quad \text{as } x \rightarrow 3 \Rightarrow$$

$$x(x-3) \rightarrow 0 \quad \text{i.e. } \theta \rightarrow 0.$$

$$= \left[\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \right] \left[\frac{3}{6} \right]$$

$$= 1 \times \frac{1}{2} = \frac{1}{2}$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 1} \frac{(x^2 - x) \sin(x-1)}{x^2 - 2x + 1}$$

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 1} \frac{(x^2 - x) \sin(x-1)}{x^2 - 2x + 1} \\ &= \lim_{x \rightarrow 1} \frac{x(x-1) \sin(x-1)}{(x-1)(x+1)} \\ &= \lim_{x \rightarrow 1} \frac{x \sin(x-1)}{(x+1)} \\ &= \lim_{x \rightarrow 1} (x) \cdot \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x+1)} \\ &= (1) \cdot \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x+1)} \end{aligned}$$

Let $x-1=\theta$, when $x \rightarrow 1 \Rightarrow \theta \rightarrow 0$.

$$= \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

الحل:
بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin^2(x)}{x^2} \dots\dots\dots [1 - \cos 2x = 2 \cos^2 x] \\ &= 2 \lim_{x \rightarrow 0} \left(\frac{\sin(x)}{x} \right)^2 \\ &= 2 [1]^2 = 2 \end{aligned}$$

مثال:

أحسب النهاية

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}$$

الحل:
بوضع

$$L = \lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}$$

(using $\sin \alpha - \sin \beta$)

$$= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\therefore L = \lim_{x \rightarrow a} \frac{2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)}{(x - a)}$$

$$= \lim_{x \rightarrow a} \frac{2 \cos\left(\frac{x + a}{2}\right) \sin\left(\frac{x - a}{2}\right)}{(x - a)}$$

$$= \lim_{x \rightarrow a} 2 \cos\left(\frac{x + a}{2}\right) \frac{\sin\left(\frac{x - a}{2}\right)}{\left(\frac{x - a}{2}\right)} \cdot \frac{1}{2}$$

put $\frac{x - a}{2} = \theta$, when $x \rightarrow a, \Rightarrow \theta \rightarrow 0$

$$= 2 \cos(a) \left[\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{(\theta)} \cdot \frac{1}{2} \right]$$

$$= 2 \cos(a) \left[1 \times \frac{1}{2} \right] = \cos(a)$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{\sin(a+x) + \sin(a-x)}{x \sin x}$$

الحل: بوضع

$$\begin{aligned}
L &= \lim_{x \rightarrow 0} \frac{\sin(a+x) + \sin(a-x) - 2 \sin a}{x \sin x} \\
&= \lim_{x \rightarrow 0} \frac{\sin a \cos x + \cos a \sin x + \sin a \cos x - \cos a \sin x - 2 \sin a - 2 \sin x}{x \sin x} \\
&= \lim_{x \rightarrow 0} \frac{2 \sin a \cos x - \cos a \sin x - 2 \sin a}{x \sin x} \\
&= -2 \sin a \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x} \\
&= -2 \sin a \lim_{x \rightarrow 0} \frac{2 \sin^2(x/2)}{x \sin x} \\
&= -2 \sin a \cdot 2 \lim_{x \rightarrow 0} \left[\frac{2 \sin^2(x/2)}{x^2} \right] \left[\frac{x^2}{\sin x} \right] \\
&= -4 \sin a \cdot \left[\lim_{x \rightarrow 0} \frac{2 \sin^2(x/2)}{x^2} \cdot \frac{1}{2} \right] \lim_{x \rightarrow 0} \left[\frac{x}{\sin x} \right] \\
&= -4 \sin a \cdot \left[\frac{1}{4} \right] [1] \\
&= -\sin a
\end{aligned}$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{\sec x - 1}{x^2}$$

الحل: بوضع

$$\because \sec x = 1 / \cos x \Rightarrow$$

$$\begin{aligned}
L &= \lim_{x \rightarrow 0} \frac{\frac{1}{\cos x} - 1}{x^2} \\
&= \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \cos x} \\
&= \lim_{x \rightarrow 0} \left[\frac{1}{\cos x} \right] \left[\frac{1 - \cos x}{x} \right] \\
&= \left[\lim_{x \rightarrow 0} \frac{1}{\cos x} \right] \cdot \left[\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \right] \\
&= [1] \cdot \left[\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \right] \\
&\therefore \frac{1 - \cos x}{x} = \frac{0}{0} \Rightarrow \\
&= [1] \cdot \left[\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \right] \cdot \left[\lim_{x \rightarrow 0} x \right] \\
&= [1] \cdot \left[\lim_{x \rightarrow 0} \frac{1 - \sin^2 x/2}{(x/2)^2} \cdot \frac{1}{4} \right] \cdot [0] \\
&= [1] \cdot [2] \left[\lim_{x \rightarrow 0} \frac{\sin^2 x/2}{(x/2)^2} \right]^2 \cdot \frac{1}{4} \cdot [0] \\
&= \frac{1}{2} [1]^2 \cdot [0] = 0
\end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

مثال: أحسب النهاية

الحل: بوضع

$$\begin{aligned}
L &= \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} \\
&= \lim_{x \rightarrow 0} \frac{1}{x^3} \left[\frac{\sin x}{\cos x} - \sin x \right] \\
&= \lim_{x \rightarrow 0} \frac{1}{x^3} \left[\frac{\sin x - \sin x \cos x}{\cos x} \right] \\
&= \lim_{x \rightarrow 0} \frac{1}{x^3} \left[\frac{\sin x(1 - \cos x)}{\cos x} \right] \\
&= \lim_{x \rightarrow 0} \left[\frac{\tan x(1 - \cos x)}{x^3} \right] \\
&= \lim_{x \rightarrow 0} \left[\frac{\tan x}{x} \right] \cdot \left[\frac{1 - \cos x^2}{x^2} \right] \cdot \left[\frac{1}{1 + \cos x} \right] \\
L &= \left[\lim_{x \rightarrow 0} \frac{\tan x}{x} \right] \cdot \left[\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \right] \cdot \left[\lim_{x \rightarrow 0} \frac{1}{1 + \cos x} \right] \\
&= \left[\lim_{x \rightarrow 0} \frac{\tan x}{x} \right] \cdot \left[\lim_{x \rightarrow 0} \frac{\sin x}{x} \right]^2 \cdot \left[\lim_{x \rightarrow 0} \frac{1}{1 + \cos x} \right] \\
&= 1 \cdot [1]^2 \cdot \left[\frac{1}{1 + \cos 0} \right] \\
&= 1 \cdot [1]^2 \cdot \left[\frac{1}{1 + 1} \right] = \frac{1}{2}
\end{aligned}$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3}$$

الحل: بوضع

$$\begin{aligned}
L &= \lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3} \\
&= \lim_{x \rightarrow 0} \frac{3 \sin x (3 \sin x - 4 \sin^3 x)}{x^3} \\
&= \lim_{x \rightarrow 0} \frac{4 \sin^3 x}{x^3} \\
&= \lim_{x \rightarrow 0} 4 \left(\frac{\sin x}{x} \right)^3 \\
&= 4 \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^3 \\
&= 4 \cdot 1 = 4
\end{aligned}$$

Exponential Limits Problems

امثلة محلولة على نهاية الدوال الاسية

$$\text{If } a_n = \left(1 + \frac{1}{n}\right)^n \Rightarrow \lim_{h \rightarrow \infty} \left(1 + \frac{k}{n}\right)^n = e^k,$$

if we put $\frac{1}{n} = \alpha \Rightarrow$ as $n \rightarrow \infty$, $\alpha \rightarrow 0$

$$\therefore \lim_{\alpha \rightarrow 0} (1 + \alpha)^{1/\alpha} = e,$$

$$\lim_{\alpha \rightarrow 0} (1 + k\alpha)^{1/\alpha} = e^k$$

وبالتالى يمكن اثبات ان:

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1,$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a, \quad (a > 0)$$

$$\lim_{x \rightarrow 0} \frac{a^{4x} - 1}{x} = \lim_{x \rightarrow 0} \frac{a^{4x} - 1}{4x} \times 4 = \log a \times 4$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{3^{2x} - 1}{\sin x}$$

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \frac{3^{2x} - 1}{\sin x} \\ &= \lim_{x \rightarrow 0} \frac{3^{2x} - 1}{x} \times \frac{x}{\sin x} \\ &= \lim_{x \rightarrow 0} \frac{3^{2x} - 1}{x} \times 2 \times \frac{x}{\sin x} \\ &= 2 \left(\lim_{x \rightarrow 0} \frac{3^{2x} - 1}{x} \right) \left(\lim_{x \rightarrow 0} \frac{x}{\sin x} \right) \\ &= 2(\log 3)(1) \dots \left[\because x \rightarrow 0, \Rightarrow 2x \rightarrow 0, \text{ Let } \theta = 2x \Rightarrow \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} \right] \\ &= \log 3^2 = \log 9 \end{aligned}$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}, \quad a > 0, \quad b > 0$$

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \frac{a^x - b^x}{x} \\ &= \lim_{x \rightarrow 0} \frac{a^x - 1 - b^x + 1}{x} \\ &= \lim_{x \rightarrow 0} \frac{a^x - 1}{x} - \left(\lim_{x \rightarrow 0} \frac{b^x - 1}{x} \right) \\ &= (\log a) - (\log b) \\ &= \log(a/b) \end{aligned}$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$$

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \frac{6^x - 3^x - 2^x + 1}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{(2 \times 3)^x - 3^x - 2^x + 1}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{(2)^x (3)^x - 3^x - 2^x + 1}{x^2} \end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{(3)^x (2^x - 1) - (2^x - 1)}{x^2} \\
&= \lim_{x \rightarrow 0} \frac{(3^x - 1) - (2^x - 1)}{x^2} \\
&= \left(\lim_{x \rightarrow 0} \frac{3^x - 1}{x} \right) \left(\lim_{x \rightarrow 0} \frac{2^x - 1}{x} \right) \\
&= (\log 3)(\log 2)
\end{aligned}$$

مثال: أحسب النهاية

$$\lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$$

الحل: بوضع

$$\begin{aligned}
L &= \lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2} = \lim_{x \rightarrow 0} \frac{a^x + 1/a^x - 2}{x^2} \\
&= \lim_{x \rightarrow 0} \frac{(a^x)^2 - 2(a^x) + 1}{(a^x)x^2} \\
&= \left(\lim_{x \rightarrow 0} \frac{1}{(a^x)} \right) \left(\lim_{x \rightarrow 0} \frac{(a^x)^2 - 2(a^x) + 1}{(a^x)x^2} \right) \\
&= \left(\lim_{x \rightarrow 0} \frac{a^x - 1}{x} \right)^2 \dots\dots [x \rightarrow 0, \therefore x \neq 0] \\
&= \frac{1}{1} (\log a)^2 = (\log a)^2
\end{aligned}$$

$$\lim_{x \rightarrow 0} (1 + 2x)^{5/x}$$

مثال: أحسب النهاية

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 0} (1 + 2x)^{5/x} \\ &= \lim_{x \rightarrow 0} \left[(1 + 2x)^{1/x} \right]^5 \\ &= (e^2)^5 \\ &= e^{10} \end{aligned}$$

$$\lim_{x \rightarrow 0} \left(\frac{1 + 7x}{1 - 9x} \right)^{1/x}$$

مثال: أحسب النهاية

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 0} \left(\frac{1 + 7x}{1 - 9x} \right)^{1/x} \\ &= \lim_{x \rightarrow 0} \frac{(1 + 7x)^{1/x}}{1 + (-9)x^{1/x}} \\ &= \frac{\lim_{x \rightarrow 0} (1 + 7x)^{1/x}}{\lim_{x \rightarrow 0} (1 + (-9)x^{1/x})} \\ &= \left(\frac{e^7}{e^9} \right) = e^{7+9} = e^{16} \end{aligned}$$

$$\lim_{x \rightarrow 0} (x)^{\frac{1}{x-1}}$$

مثال: أحسب النهاية

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow 0} (x)^{\frac{1}{x-1}} \dots \left[\text{it is } 1^\infty \text{ form} \right] \\ &= \text{put } x-1 = h, \text{ as } x \rightarrow 1, h \rightarrow 0 \\ &= \lim_{h \rightarrow 0} (1+h)^{\frac{1}{h}} \\ &= e \end{aligned}$$

$$\lim_{x \rightarrow e} \frac{\log x - 1}{x - e}$$

مثال: أحسب النهاية

الحل: بوضع

$$\begin{aligned} L &= \lim_{x \rightarrow e} \frac{\log x - 1}{x - e} \dots \left[\text{it is } \frac{0}{0} \text{ form} \right] \\ &= \text{put } x - e = h, \text{ as } x \rightarrow e, h \rightarrow 0, \text{ also put } 1 = \log e. \\ &= \lim_{h \rightarrow 0} \frac{\log(e+h) - \log e}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\log \frac{e+h}{e} \right) \\ &= \log \left[\lim_{h \rightarrow 0} \frac{1}{h} \left(1 + \frac{1}{e} h \right)^{1/h} \right] \\ &= \log \left[e^{1/e} \right] = \frac{1}{e} \log e = 1/e \times 1 = 1/e. \end{aligned}$$