A REVIEW OF THE PRINCIPLES OF THE ISLAMIC CALENDAR AND A PROPOSAL FOR ESTABLISHING A RELIABLE INTERNATIONAL ISLAMIC CALENDAR

Abdur Rahim Khan

Abstract

This paper reviews the geometry of crescent formation and the psychology of vision. The maximum width of the crescent is calculated, and an equal visibility criterion for clear sky conditions is developed for the determination of the start of an Islamic month. The proposed criterion accounts for the latitude and longitude of the observer, and the actual ellipticity and tilt of the lunar orbit. The concept of a date line is reviewed and an Islamic date line with global validity is proposed for the establishment of an international Islamic calendar. This paper is deliberately written in layman's terms. It is assumed that the reader knows only high school level mathematics and has no prior knowledge of astronomy.

INTRODUCTION

At this time Muslims have no definite calendar. Some of them feel that it is impossible to make one because there is something unpredictable about the motion of the moon. This causes a great deal of argument and confusion in the Muslim community and is embarrassing for all of us. While others have visited the moon and have planted their standard there, every Ramadan Muslims indulge in acrimonious debate over where it is. In reality, the calendar based on the Qur'an that was established during the days of the Prophet Muhammad is quite definite and can be used internationally if we get organized and agree upon certain conventions. Because this has not been done each community and often individuals make up their own rules resulting in an argument over what day of the month it is. Europeans went through a similar phase in the last century. As they started traveling great distances quickly, there was confusion over dates. Soon a date line was agreed upon among the leaders and the common man was merely informed after the decision. Now, because of the general availability of telecommunication, every Muslim is involved in this debate. However, the lunar cycle is not as obvious as the solar cycle of day and night. Many people do not realize that just as we cannot find whether it is day or night at our location by phoning the home country, we cannot be certain about a lunar date confirmed by another half a world away. One has to look at the local sky. Every Muslim is not an astronomer, he cannot rationally decide about such matters. His decisions are generally based on emotional and devotional reasons and are often scientifically wrong. It is essential that the community pay urgent attention to solve this problem.

SOME COMMENTS ON THE MOTION OF HEAVENLY BODIES

The relative motion of all heavenly bodies is governed by a few simple laws commonly known as Newton's laws of motion and the law of gravity. The curved path of a body through the heavens is such that gravitational force is always balanced against (i.e., equal and opposite to) the centrifugal force due to the curvature of the path and the inertial force due to the acceleration of the body. As a result, planets generally move in an elliptic orbit around the sun such that the sun remains at one of the foci of the ellipse. A circular orbit is just a special case of an ellipse when the two foci are congruent. The two equations that have to be solved simultaneously for computing the orbit of a planet are as follows:

$$\text{Gravitational force} = G \cdot M1 \cdot M2/R^2 \quad (1)$$

$$\text{Acceleration of a body} = G \cdot M2/R^2 \quad (2)$$

Here G is the gravitational constant, M1 is the mass of the planet, M2 the mass of the sun, and R is the instantaneous distance between them.

For a two-body system the method of solution is quite simple. As the number of bodies increases it becomes increasingly difficult to solve these equations. But all this has been done by modern computers, and tables giving relative positions of bodies in our solar system are available in most libraries. To construct a calendar, all one has to do is to learn to read and use them.

THE EARTH, MOON AND SUN SYSTEM

The Qur'an says that the sun and the moon move according to a *ḥisāb*, i.e., governing laws. These laws have been known in detail for some time so that we may be confident that we know the location of a heavenly body at any given time. This is all that is needed to construct a prayer timetable and a calendar. A calendar can be viewed as a special prayer timetable that tells us when to say *tarawīḥ* or *Īd* prayers.

MOTION OF THE EARTH

Basically, the earth moves in three ways.

1. It spins on its axis to produce day and night. This takes 24 hours. Its spin axis is tilted about 23.5 degree in relation to its orbital plane.
2. It revolves around the sun in a near circular orbit. Because of the tilt of its rotational axis the sun appears to move north and south through the year. This change in the angle of incidence of light from the sun is responsible for the change of seasons. Its rotational period is about 365.25 days or roughly a year. This necessitates frequent correction in the length of the solar year to prevent the seasons from sliding through the year. This is done by means of a system of having leap years (366 day years) every four, one hundred, four hundred, and four thousand years. This also corrects for fractional number of days in a solar year.
3. A third, less known, motion of the earth is called precession. The axis of rotation describes a cone in space over a period of about 25,800 years.

MOTION OF THE MOON

The moon moves under the influence of gravity, mainly, of the sun and the earth. It is also disturbed slightly by other planets. Its orbit around the earth is only approximately elliptic because the sun disturbs it. Its orbital plane is tilted-in relation to the earth's orbital plane (ecliptic) by about 5 degrees. The moon's orbital plane is not fixed in space in relation to the stars. Even though it maintains a constant tilt in relation to the earth's orbital plane, the line of intersection of these two planes (nodes) rotates slowly in space. Because of the tilt of its orbit the moon appears to move to the north and south of the sun, and the new crescent is sometimes seen to the left and at other times to the right of the setting sun. Viewed from space the path of the moon is like a sine wave superimposed over the near circular path of the earth.

The moon produces no light of its own and shines due to sunlight. Like earth, only half the surface of the moon's sphere is illuminated by the sun at any time. Depending upon the relative position of the three bodies we can see varying portions of the illuminated side of the moon from the earth. When the moon is between the earth and the sun only the far side of the moon is illuminated and we cannot see it. This position is called astronomical new moon. When the earth is between the moon and the sun and the side of the moon facing us is illuminated by the sun, we see full moon. Viewed from the moon, the earth appears to change phases much like the moon. Figure 1 shows the sphere of the moon and the relationship of the directions of the sun and the earth at the time of first visibility of the crescent.

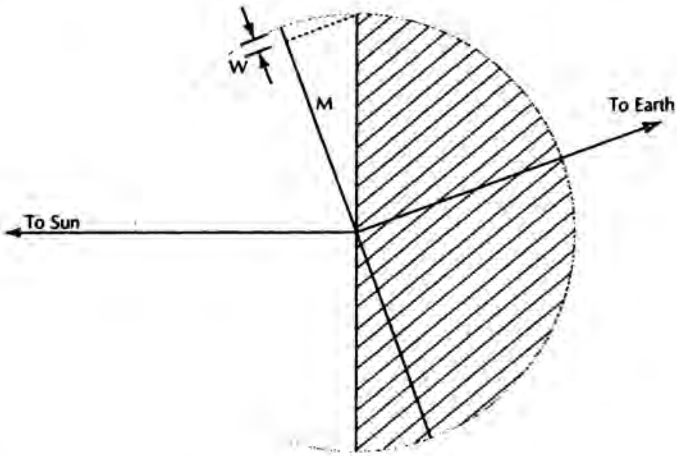

Figure 1.

From this diagram it can be seen that the width of the crescent (w) that can be seen from earth is $r \cdot (1\text{-Cos } M)$. Here r is the radius of the moon's disk and M is the angle between the lines of sight from the earth to the sun and the moon. Since the moon appears to move about 360 degrees around the earth in relation to the sun in one month (about 29.5 days) its apparent daily motion is about 12.2 degrees on the average. Because of the ellipticity of the moon's orbit this number varies from 11.5 to 13.1 degrees a day. Earliest visibility is possible when the moon is closest to the earth and the angle from the sun is changing fastest. Therefore, the greatest angle $M = 13.1 \cdot A/24$ degrees. Here A is the age of the moon or the time from astronomical new moon in hours. In radians $M = 13.1 \cdot A \cdot 3.14/24 \cdot 180$ or about

0.00952·A. Since Cos $M = 1 - M^2/2$ when M is small, the width of the crescent of age A hours as seen from the earth can be approximated as follows.

$$\text{Width} = r \cdot (0.00952 \cdot A)^2/2 \qquad (3)$$

Since the minimum distance of the moon from the earth is about 357,000 km and r = 1,738 km, the angle subtended by the widest part of the crescent on the eye of an observer on earth is its width divided by the distance. Therefore,

$$\text{Angle} = 1738 \cdot (0.00952 \cdot A)^2/2 \cdot 357000 \text{ radians} \qquad (4)$$

$$= 0.045 \cdot A^2 \text{ seconds of arc} \qquad (5)$$

Hence it is seen that the width of the young crescent is proportional to the square of its age. The constant 0.045 is just an estimate, it depends on the distance of the moon from the earth. Since the resolving power of the human eye is about 20 seconds of arc (1 second of arc is 1/3600 degrees) the approximate age of the crescent for visibility can be estimated from equation (5) as follows:

$$20 = 0.045 \cdot A^2 \qquad (6)$$

or $$A = 21 \text{ hours.}$$

This is the basis for the much talked about 20- or 22-hour rule for the earliest visibility of the crescent. In the tropics, if the age is much more than this, one can be sure that it will be seen, and if it is much less than this we can safely say that it will not be seen. Because of a number of other factors, visibility of the crescent actually varies with a much higher power of age than the square law derived from geometry only. They are discussed later.

MOTION OF THE SUN

The sun also moves in relation to other stars in our galaxy, but the entire planetary system moves with it. Since our interest is limited to relative motion of the earth-moon-sun system we do not have to concern ourselves with the motion relative to other stars.

ISLAMIC CALENDAR CONVENTIONS: RELIGIOUS REQUIREMENTS

The Qur'an clearly says that the sun and the moon move according to laws and are used for the reckoning of time. There has always been a tendency among common people of all faiths to ascribe some mystical meaning or significance to their appearance and motion. The Qur'an does not support such a view—to the Muslim, they are nature's clocks.

Islamic conventions about time keeping are different from those in common use in the western world today. For example, to a Muslim a day begins at sunset, not midnight. A week, month, and year are similarly affected.

The 12 Islamic months are not of fixed duration; for example, Muharram can sometimes be 29 days and sometimes 30 days. The month starts with the first visibility of the new moon and not with the birth of the moon or with the astronomical new moon as in some other calendars.

Calendars can be based on either the motion of the moon, the sun, or both. Most lunar calendars are luni-solar, that is, although they count a month based on the lunar cycle, they introduce some corrections to keep their calendar synchronized with the seasons that depend on the solar cycle. Usually, this is done by adding a 13th month every few years, and is known as intercalation. The Qur'an forbids intercalation, and clearly states that a year has 12 months. Since a lunar month on the average has about 29.5 days, the Islamic year has only 354±1 days. This injunction leaves the Islamic year about 11.25 days shorter than the average solar year. For this reason our observances move through the seasons and come about 11 days earlier in each succeeding year.

THE PSYCHOLOGY OF VISION: HOW DO WE SEE THINGS?

Visibility is not only the property of an object, it is the result of the interaction between an object and its surroundings with an observer in the presence of a light source. We see things when a sufficient amount of light bounced off an object is able to reach our eyes. Based on everyday experience we can say that an object becomes visible if the following two conditions are met:

- It is big enough—very small things cannot be seen; and

- Its illumination is significantly different from its surroundings. For example, white on white is invisible, black on white is very visible, and gray on white is less visible.

Therefore, visibility to a man with good eyesight depends on contrast and size. As discussed earlier, the width of the crescent depends on its age. Because an older crescent remains above the horizon longer after sunset its contrast in relation to the darker sky of later evening also increases. This further aids visibility.

ACUITY OF VISION

As indicated earlier, a person with normal vision can resolve down to about 20 seconds of arc. That is, he can recognize those visual details in a scene that subtend an angle greater than 20 seconds of arc at his eye if sufficient contrast is present. Objects smaller than this appear to be of this size but at a reduced contrast. If a finer (less that 20 seconds of arc wide) black line is drawn on white paper, it appears wider but gray to us. Similarly, a source of light smaller than this number appears bigger but less bright to us.

The approximate criterion of visibility based on size alone was previously developed under the topic "Motion of the Moon." For completeness, that is, to get the precise theoretical condition for visibility it has to be combined with the contrast requirement. I say theoretical because even after we account for standard atmospheric conditions there is no guarantee of sighting because actual atmospheric conditions are variable and unpredictable.

DEFINITION OF CONTRAST AND THINGS THAT AFFECT IT

Contrast between two parts of a scene is defined as the ratio between the difference of their illumination and the sum of their illumination. Symbolicly, we may write it in the following form:

$$\text{Contrast} = (IMAX - IMIN)/(\text{IMAX} + IMIN) \qquad (7)$$

Here *IMAX* and *IMIN* are the maximum and minimum illuminations in a scene.

Our atmosphere greatly reduces contrast because of two effects. Without an atmosphere the sky would appear black, i.e., *IMIN* would be zero, and *IMAX* would be much higher because all the light from the moon would reach us without being absorbed or scattered by the atmosphere.

Atmospheric absorption and scattering characteristics are highly variable and unpredictable. While it is possible to characterize for an existent atmosphere very accurately, it is not possible to predict what a future atmosphere will be like because, besides the innumerable unpredictable natural phenomena, there are also affects caused by human activity. Minute and often imperceptible changes in the atmosphere can affect contrast immensely. For example, it is a matter of common experience that the sunset looks different every day even when the atmosphere appears clear. The following section analyses the effect of atmospheric changes in a more quantitative way.

Absorption and Scattering by the Atmosphere

The equations of atmospheric absorption can be developed easily with the help of Figure 2 below.

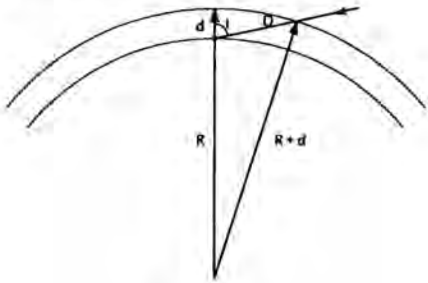

Figure 2.

If R is the radius of the earth and d is the vertical depth of the atmosphere, the distance, D, a slanting beam of light has to travel through the atmosphere before reaching ground can be calculated from the cosine law of triangles as follows:

$$R^2 + D^2 + 2R \cdot D \cdot \mathrm{Cos}(I) = (R + d)^2 \qquad (8)$$

Here I is the angle of incidence from the vertical. Solving for D gives:

$$D = - R \cdot \mathrm{Cos}(I) + (R^2 \cdot (\mathrm{COS}(I))^2 + d^2 + 2R \cdot d)^{1/2} \qquad (9)$$

Table 1 (placed at the end of this paper) gives values of D for various values of I. If the vertical depth of atmosphere d is assumed to be 5 miles, at sunset the light would have to travel about 197 miles through our atmosphere and much of it would get absorbed or scattered. It is for this reason that the sun appears so dull at sunset. The magnitude of this absorption can be calculated from the standard logarithmic law, i.e., each thickness of atmosphere reduces transmission by a fixed ratio. For example, if we assume that only 70% of sun's light reaches us when the sun is overhead, only 0.7·0.7, i.e. 49% of it will reach us in case of a slanting entry when $D = 2d$. At the time of sunset when D is about $40 \cdot d$, the fraction that will be able to reach us will be only 0.7 to the power 40 = 6E-7, i.e., only 0.6 parts per million will reach us. If the atmospheric absorption is changed only 10% such that 77% of light is able to reach us for vertical incidence, the amount reaching us at sunset will be 0.77 to the power 40 = 2.88E-5. This is 48 times more than the previous case. It should now be obvious why the sunset looks so different every day even though the sky appears clear. Table 2 lists values of transmission through the atmosphere for various angles of shallow incidence and atmospheric absorption coefficients.

Frans Bruin has reported values of brightness for various altitudes of full moon.[1] They agree well with my theoretical development for estimating transmission through the atmosphere.

Illumination of the Sky after Sunset

This is best determined experimentally. Frans Bruin has reported values of illumination for various solar dip angles.[2] If necessary, fresh measurements can be taken with better instruments. It is necessary that this function be agreed upon for the development of the calendar.

Actual Contrast of the Moon as Seen from Earth

Illumination of the moon as seen from the earth is the sum of transmitted light of the moon and the illumination of the sky. Therefore:

$$IMAX = IMOON \cdot F(I) + ISKY. \qquad (10)$$

Here *IMOON* is the original illumination of the moon before reaching the atmosphere, $F(I)$ is the transmission function of angle of incidence I (as given in Table 2 or whatever function is ultimately chosen), and *ISKY* is the illumination of the atmosphere.

$$IMIN = ISKY \qquad (11)$$

1. Frans Bruin, "The First Visibility of the Lunar Crescent." *Vistas in Astronomy* (Great Britain: Pergamon Press) 21 (1977): 331–358.

2. Ibid.

Therefore, contrast of the moon as seen from the earth is given by the following expression:

$$\text{Contrast} = IMOON{\cdot}F(I)/(IMOON{\cdot}F(I) + 2{\cdot}ISKY). \qquad (12)$$

IMOON·F(I) can be measured, once *F(I)* and *ISKY* are standardized, and contrast can be calculated for all configurations of the sun and the moon near sunset.

At the time of sunset the illumination of the sky is too high. From equation (12) it can be seen that for high *ISKY* the contrast is low and the moon cannot be seen. As the evening progresses, *ISKY* reduces and the contrast improves. Later as the moon approaches the horizon, its light is increasingly absorbed by the atmosphere and the portion reaching us declines very quickly. Hence the contrast starts reducing after reaching a maximum value, and continues to do so until the moon sets.

Combining Size and Contrast Data

Experimentally determined values of minimum contrast required to make an object just visible are available in at least two forms. One experiment involves the use of disks of various sizes and shades, and the other is based on the use of sinusoidally varying grating. Both produce data that can be reduced to a simple curve of size vs. minimum contrast required for visibility. Once the size of the crescent is known, the condition for visibility is reduced to comparing this number with the highest contrast achieved during the evening as determined from equation (12). If the achieved contrast is greater than the minimum required, the moon should be declared visible.

CAN THE VARIATION IN THE ATMOSPHERIC CONDITION BE ALLOWED TO INFLUENCE THE CALENDAR?

As indicated earlier weather and atmospheric phenomena cannot be predicted at the present time with any degree of certainty. Even if later technology can predict them with greater accuracy, the noncyclical nature of changes in weather will remain. Because of the unpredictability of weather, much confusion is introduced by some people on religious grounds. Comments like "We told you the moon is unpredictable" are often heard. Some people insist that we have to delay the arrival of a month in case of clouds. Such an interpretation of certain hadith is not only contrary to the Qur'an but also to common sense. Surely, it would be silly for Muslims to have a calendar based on clouds. If no other information is available to determine the condition of the moon, it is prudent to err on the safe side. But it is wrong to allow the randomness of clouds to take precedence over the firmly established and known cyclical motion of the moon. If we believe in the Qur'an and accept that the moon is for the reckoning of time, it would be wrong to introduce the absence or appearance of clouds and fog as a necessary element in time keeping. For these reasons I firmly believe that contrast calculations should be based on some standard clear atmosphere only.

DEVELOPMENT OF THE INTERNATIONAL LUNE DATE LINE

Just as there is a solar date line (180 degrees longitude) to mark the start of a date on a solar calendar, a lunar date line can be constructed to indicate the start of a lunar date. Because an Islamic month begins in the evening with the first visibility of the crescent, these points of first visibility can be considered to constitute the lunar date line.

For a given solar dip angle and angular separation between the moon and the sun, the moon appears higher above the horizon at sunset near the equator than at high latitudes. For this reason higher contrast is achieved in the tropical region. As we move to higher latitudes, because of reduced contrast, a bigger crescent is required for visibility and visibility is delayed. For this reason the line of first visibility appears like a parabola enlarging towards the west.

NEED FOR A MODIFIED DATE LINE

Since the earth's spin axis is tilted to an angle of 23.5 degrees, and the moon's orbital plane is further tilted about 5 degrees to the earth's orbital plane, the moon cannot be expected to come above the horizon every month at latitudes greater than approximately $90 - (23.5 + 5) = 61.5$ degrees. Further, in such places the sun may appear higher than the moon, and visibility may be impossible. I say "may be" because the conditions vary from month to month. Even at somewhat lower latitudes where visibility may be possible it may be delayed so much that the months become shorter and longer than 29 and 30 days. Obviously, such a situation is not acceptable and some other criteria are required to determine the start of a lunar month at high latitude. In practice, the problem starts near 50 degrees latitude. For example, Muharram 1408 becomes 31 days near 50 degrees North latitude. For this reason I propose that the local visibility requirement be abandoned at latitudes higher than 45 degrees (Toronto is at 44 degrees), and the lines of longitude be treated as the Islamic date lines from this latitude to the poles. Once we adopt this convention a lunar calendar can be constructed for all places on earth; otherwise, much of Canada will be without an Islamic calendar. When Ramadan and the two *Īds* take place in the Winter, confusion will increase because the crescent will not be seen on time over much of Canada even in clear weather.

APPROXIMATIONS FOR SIMPLIFYING THE CALCULATIONS

The exact method described above is too complex for people not trained in astronomy and mathematics. The labor and skill required for constructing such a calendar can be justified only if the product is widely used around the world. Further, even if the method proposed here ultimately gains general acceptance it may take many years to accomplish. In the interim a simpler method that can be used to construct a local calendar quickly is needed for use by communities who choose to implement it on their own. The present state of utter confusion is too painful and embarrassing for most of us. Even the 20-hour visibility criterion is more reliable and accurate than what we have been able to achieve by reliance on rumors of sighting from far away places.

Once we realize that for any locale the average age for sighting is about 32 hours, and the range is from about 20 hours to 20 hours plus a day, i.e., 44 hours, it becomes obvious that there can only be a few months for which accurate calculation may be required because the age of the crescent is close to 20 hours. For most months there is no doubt about visibility because the age is considerably more that 20 hours. For those months where the age is close to 20 hours the question of contrast can be answered quite reliably if the elevation of the moon above the horizon is known at sunset, or the time difference between sunset and moonset is known. If the moon is more than 20 hours old and remains above the horizon for more than 30 minutes after sunset, it is our experience that it can be seen. In Toronto we have relied on the 20-hour age in combination with minimum 30-minute time difference between sunset and moonset as a criterion for visibility for many years. Based on more than 10 years of experience we can report that the method has never failed. The Hilal Committee of

Toronto and Vicinity uses this method for developing the calendar for local use.

THE INTERNATIONAL ISLAMIC CALENDAR

There are two practical difficulties with the use of the precise date line developed here.

1. The Islamic date line is different every month. In general it passes through land masses and may divide countries and even large cities into regions of two different dates. This is very inconvenient and is not likely to be accepted by people.
2. Because the international date line is fixed and the Islamic date line revolves around the globe, in general an Islamic date straddles two Gregorian dates and vice versa. Unless some other adjustment is made in the Islamic date line it is impossible to ensure observance of any Islamic festival on one day throughout the world.

If we adopt a convention that the Islamic date line will be moved to the nearest ocean (Atlantic or Pacific as the case may be) the first problem will be solved almost completely, and on the average, half the observances will be on the same day throughout the world. I personally do not consider observance on one Gregorian date very important, but because certain groups feel that this is an essential element for Muslim unity, I have suggested a systematic way of achieving this condition. Other schemes proposed by fiqh committees are not worthy of discussion.

Table 1.

Angle of Incedence	Length of Transmission	Fraction Transmitted
0	5.0	.7
2	5.00317	.699842
4	5.01221	.699391
6	5.02734	.698636
8	5.04907	.697554
10	5.0769	.69617
12	5.11133	.69463
14	5.15283	.69241
16	5.20117	.690026
18	5.25684	.687292
20	5.32031	.684187
22	5.39185	.680704
24	5.47217	.676815
26	5.56152	.672515
28	5.66138	.667742
30	5.77124	.662529
32	5.89356	.656773
34	6.02832	.65049
36	6.17676	.643638
38	6.34106	.636138
40	6.52222	.627971
42	6.72241	.619066
44	6.94409	.609354
46	7.18994	.59876
48	7.46338	.587194
50	7.76782	.574579
52	8.10816	.560798
54	8.49097	.545691
56	8.92285	.529135
58	9.4126	.510968
60	9.97217	.490974
62	10.616	.468936
64	11.363	.444599
66	12.2385	.417682
68	13.2771	.387856
70	14.5264	.354787
72	16.0547	.318141
74	17.9637	.277636
76	20.4105	.233172
78	23.6506	.185052
80	28.1293	.134444
82	34.6891	8.42012E-02
84	45.1112	4.00348E-02
86	63.7472	1.05947E-02
88	103.036	6.42559E-04
90	196.903	7.9411E-07

Table 2.

Fraction of Light Transmitted through the Atmsosphere at Different Angles

0°	84°	86°	88°	90°
.6	9.96388E-03	1.48441E-03	2.68121E-05	1.83435E-09
.605	1.07386E-02	1.65008E-03	3.18128E-05	2.5434E-09
.61	1.15663E-02	1.83264E-03	3.76931E-05	3.51705E-09
.615	1.24504E-02	2.03365E-03	4.45984E-05	4.85058E-09
.62	.013394	2.25481E-03	5.2697E-05	6.67233E-09
.625	1.44006E-02	2.49796E-03	6.21828E-05	9.15482E-09
.63	.015474	2.76506E-03	7.32794E-05	1.25293E-08
.635	.016618	3.05827E-03	8.62442E-05	1.71052E-08
.64	1.78365E-02	3.3799E-03	1.01373E-04	2.32952E-08
.645	1.91339E-02	3.73244E-03	1.19006E-04	3.16492E-08
.65	2.05145E-02	4.11861E-03	1.39533E-04	4.28974E-08
.655	2.19829E-02	4.54129E-03	1.63402E-04	5.80079E-08
.66	2.35441E-02	5.00364E-03	1.91124E-04	7.82612E-08
.665	2.52032E-02	5.50903E-03	2.23285E-04	1.05347E-07
.67	2.69653E-02	6.0611E-03	2.60554E-04	1.41493E-07
.675	2.88362E-02	6.66375E-03	3.03695E-04	1.89624E-07
.68	3.08216E-02	7.3212E-03	3.53579E-04	2.53579E-07
.685	3.29277E-02	8.03798E-03	4.11198E-04	3.38383E-07
.69	3.51608E-02	8.81893E-03	4.77683E-04	4.50601E-07
.695	3.75275E-02	9.66929E-03	5.54316E-04	5.98795E-07
.7	4.00348E-02	1.05946E-02	6.42558E-04	7.94107E-07
.705	.04269	.011601	7.44064E-04	1.05101E-06
.71	4.55006E-02	1.26948E-02	8.60713E-04	1.38827E-06
.715	4.84746E-02	.013883	9.94633E-04	1.83018E-06
.72	5.16202E-02	1.51729E-02	1.14823E-03	2.40812E-06
.725	.054946	1.65725E-02	1.32423E-03	3.16254E-06
.73	.058461	1.80902E-02	1.52571E-03	4.14554E-06
.735	6.21745E-02	.019735	1.75615E-03	5.42405E-06
.74	6.60963E-02	2.15168E-02	2.01947E-03	7.08394E-06
.745	7.02366E-02	2.34457E-02	2.32009E-03	9.23518E-06
.75	7.46058E-02	2.55329E-02	2.66298E-03	1.20184E-05
.755	7.92151E-02	2.77902E-02	3.05375E-03	1.5613E-05
.76	8.40759E-02	3.02301E-02	3.4987E-03	2.02477E-05
.765	8.92001E-02	3.28661E-02	4.00491E-03	2.62135E-05
.77	9.46002E-02	3.57124E-02	4.58033E-03	3.388E-05
.775	.100289	3.87845E-02	5.23387E-03	4.3716E-05
.78	.10628	4.20984E-02	5.97553E-03	5.63153E-05
.785	.112587	4.56716E-02	6.81651E-03	7.24285E-05
.79	.119225	4.95224E-02	7.76936E-03	9.30034E-05
.795	.126208	5.36705E-02	8.84809E-03	1.19235E-04
.8	.133553	5.81367E-02	1.00684E-02	1.52627E-04